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MTH 111 Math foe Architects Spring 2014, 1–4

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MTH 111, Math. for the Architects, Exam I, Spring 2014

Ayman Badawi

(Each question = 10 points, total points 100 points)

QUESTION 1. Find an equation of the ellipse with the vertices $V_1(4,3)$, $V_2(1,7)$, and $V_3(-2,3)$. Find the constant k. Find the foci. Make a rough sketch of such ellipse.

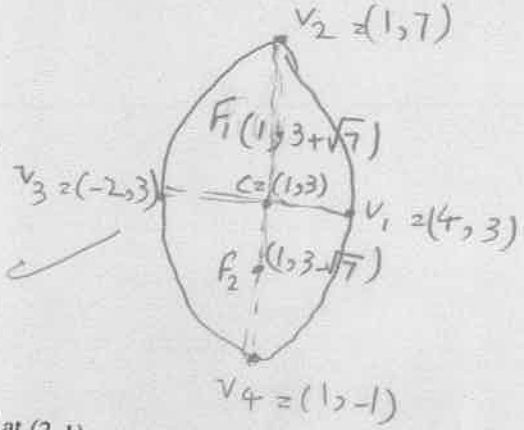
$c = \left(\frac{4-2}{2}, 3\right) \rightarrow \boxed{c = (1,3)}$: because c is midle of the V_1 and V_3

$\boxed{L \text{ axis} = 7-3 = 4}$, $\boxed{S \text{ axis} = 4-1 = 3}$ $\rightarrow V_4 = (1,3-4) \rightarrow \boxed{V_4 = (1,-1)}$

$CF = \sqrt{L^2 - S^2} \rightarrow CF = \sqrt{16-9} = \boxed{\sqrt{7}} \rightarrow F_1 = (1, 3+\sqrt{7})$, $F_2 = (1, 3-\sqrt{7})$

$k = 2L \rightarrow k = 2 \times 4 = 8$

$$\rightarrow \boxed{\frac{(x-1)^2}{4^2} + \frac{(y-3)^2}{3^2} = 1}$$



QUESTION 2. Find an equation of the hyperbola that is centered at (2, 1) and with constant $k = 6$ such that (2, 6) is one of the foci. Find the second foci, find the vertices, and make a rough sketch of such hyperbola.

$C = (2,1)$

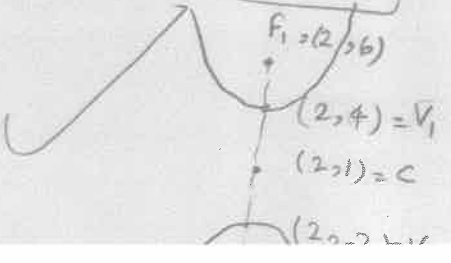
$k = 6 \rightarrow 2M = 6 \rightarrow \boxed{M = 3}$

$|FC| = 6 - 1 = 5 \Rightarrow N^2 = |FC|^2 - M^2 \rightarrow N^2 = 25 - 9 \rightarrow N^2 = 16 \rightarrow \boxed{N = 4}$

$$\boxed{\frac{(y-1)^2}{3^2} - \frac{(x-2)^2}{4^2} = 1}$$

$F_1 = (2,6)$, $C = (2,1)$, $FC = 5$ $\Big\}$ $F_2 = (2, 1-5) = \boxed{(2,-4)}$

$C = (2,1)$, $M = 3$ $\Big\}$ $V_1 = (2, 1+3) = (2,4)$, $V_2 = (2, 1-3) = (2,-2)$



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QUESTION 3. Given $x = 1$ is the directrix line of the parabola that passes through the point $(6, 5)$ and the line $y = 2$ passes through the vertex of the parabola. Find the vertex, the focus, and make a rough sketch of such parabola. Then find an equation of the parabola.

Good!!

$Q=(6,5) \rightarrow |QL| = |QF|$

$\rightarrow |QL| = 6-1 = 5$

$F=(m,2) \rightarrow |QF| = \sqrt{(6-m)^2 + (5-2)^2} = 5$

$\rightarrow (6-m)^2 + 9 = 25 \rightarrow (6-m)^2 = 16$

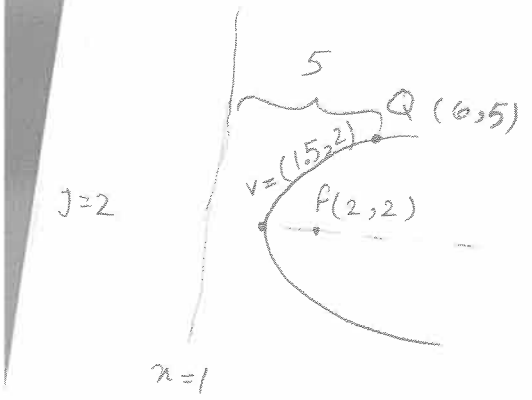
$\rightarrow 6-m = 4 \rightarrow \boxed{m=2}$

$\boxed{F=(2,2)}$ — $\boxed{\text{Vertex} = \left(\frac{1+2}{2}, 2\right) = (1.5, 2)}$

$\rightarrow \boxed{P = 2 - 1.5 = 0.5}$

$\Rightarrow (y-2)^2 = 4(0.5)(x-1.5)$

$= (y-2)^2 = 2(x-1.5)$



QUESTION 4. Find the directrix, the focus, and the vertex of the parabola $y = 0.5(x+5)^2 + 4$

$0.5(x+5)^2 + 4 = y \rightarrow (x+5)^2 = 2y - 8 \rightarrow (x+5)^2 = 2(y-4)$

$4P = 2 \rightarrow \boxed{P = \frac{1}{2}}$, $\boxed{\text{Vertex} = (-5, 4)}$

$\rightarrow \boxed{F = (-5, 4 + \frac{1}{2})}$

$\boxed{y = 4 - \frac{1}{2} = \frac{7}{2}}$

$F = (-5, 4+\frac{1}{2})$

$(-5, 4)$

$y = 4 - \frac{1}{2} = \frac{7}{2}$: directrix

QUESTION 5. Find the foci, the constant k, and the vertices of the ellipse $(x+2)^2/25 + (y-3)^2/9 = 1$

$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{9} = 1 \rightarrow \begin{cases} L^2 = 25 \rightarrow L = 5 \\ S^2 = 9 \rightarrow S = 3 \end{cases}$

$\Rightarrow |CF|^2 = L^2 - S^2 = 25 - 9 = 16$

$\rightarrow \boxed{CF = 4}$

$(-2, 3)$

4

$\rightarrow F_1 = (-2+4, 3) = (2, 3)$

$F_2 = (-2-4, 3) = (-6, 3)$

$\rightarrow \boxed{k = 2 \times 5 = 10}$

$C = (-2, 3)$ $V_1 = (-2+5, 3)$, $V_4 = (-2-5, 3)$ $V_1 = (3, 3)$

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QUESTION 6. Find the center, the foci, the vertices of the hyperbola $x^2 - 2y^2 - 4y = 18$

$x^2 - 2(y^2 - 2y) = 18 \rightarrow x^2 - 2((y-1)^2 - 1) = 18 \rightarrow x^2 - 2(y-1)^2 = 16$

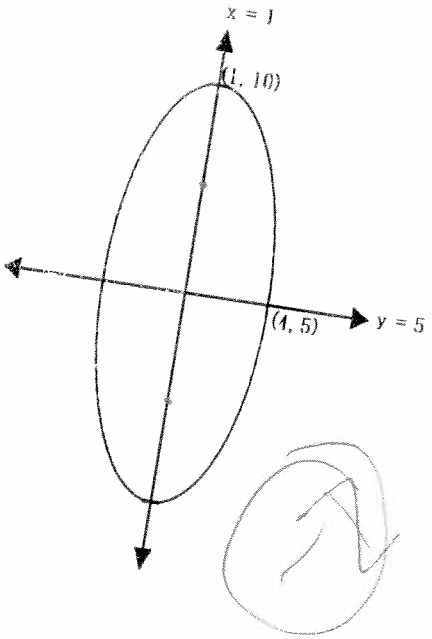
$\frac{(x-0)^2}{16} - \frac{(y-1)^2}{8} = 1 \rightarrow$ C = (0, 1) & $M^2 = 16 \Rightarrow M = 4$

$|CF|^2 - M^2 = N^2 \rightarrow |CF|^2 = 16 + 8 \Rightarrow |CF|^2 = 24 \rightarrow |CF| = \sqrt{24}$

$F_1 = (0 + \sqrt{24}, 1)$, $F_2 = (0 - \sqrt{24}, 1)$

$V_1 = (0+4, 1) = (4, 1)$, $V_2 = (0-4, 1) = (-4, 1)$

QUESTION 7. Find the foci, and the equation of the below ellipse:



$C = (1, 5) \rightarrow L = 10 - 5 = 5$

$S = 5 - 1 = 4$

$\rightarrow \frac{(x-1)^2}{4^2} + \frac{(y-5)^2}{5^2} = 1$

$L^2 = |CF|^2 - 5^2 \rightarrow |CF|^2 = 25 + 16$

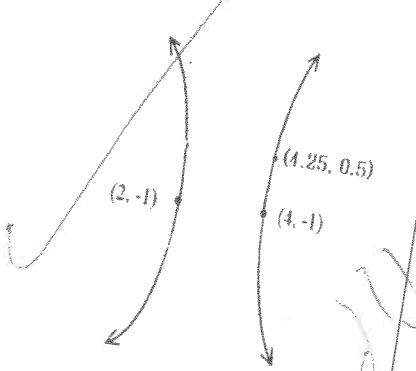
$\rightarrow |CF|^2 = 41 \rightarrow |CF| = \sqrt{41}$

$\rightarrow F_1 = (1, 5 + \sqrt{41})$, $F_2 = (1, 5 - \sqrt{41})$

Ok wrong

QUESTION 8.

Find the foci, and the equation of the below hyperbola:



$C = (\frac{4+2}{2}, -1) = (3, -1)$

$\rightarrow M = 4 - 3 = 1 \rightarrow M = 1$

$\rightarrow \frac{(x-3)^2}{1} - \frac{(y+1)^2}{N^2} = 1$

$\rightarrow \frac{(4.25-3)^2}{1} - \frac{(0.5+1)^2}{N^2} = 1 \rightarrow \frac{1.5625}{1} - \frac{2.25}{N^2} = 1$

$\rightarrow 1.5625 - 1 = \frac{2.25}{N^2} \rightarrow N^2 = \frac{2.25}{0.5625} = 4$

$\rightarrow N = 2$

Foci: $|CF|^2 - M^2 = N^2 \rightarrow |CF|^2 = 1 + 4$

$CF = \sqrt{5}$

$F_1 = (3 + \sqrt{5}, -1)$

$F_2 = (3 - \sqrt{5}, -1)$

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QUESTION 9. Find an equation of the plane P that contains the line $L: x = t, y = 1 - t, z = 2t$ and the point $Q = (1, 0, 5)$ [note that the point Q does not lie on L]

$t = 0 \rightarrow P_0 = (0, 1, 0)$; P_0 is a point on the L.

$\vec{v} : i - j + 2k$; directional vector of L.

$\vec{QP_0} : -i + j - 5k \longrightarrow \vec{QP_0} \times \vec{v} = \vec{N}$ ($\vec{N}$ is the normal vector of the plane.)

$$\Rightarrow \vec{QP_0} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -1 & 1 & -5 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 1 & -5 \end{vmatrix} i - \begin{vmatrix} 1 & 2 \\ -1 & -5 \end{vmatrix} j + \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} k = 3i - (-7j) + 3j$$

$\Rightarrow$ P: $3(x-1) + 7(y-0) = 0$

QUESTION 10. a) Find the distance between the point $Q = (2, 2, 1)$ and the plane $x + 3y + 5z = 15$

$N = i + 3j + 5k$

$$\rightarrow D = \frac{|2 + (3 \times 2) + (5 \times 1) - 15|}{\sqrt{1^2 + 3^2 + 5^2}} = \frac{|2 + 6 + 5 - 15|}{\sqrt{1 + 9 + 25}} = \frac{|-2|}{\sqrt{35}} = \frac{2}{\sqrt{35}}$$

b) The line $L_1 : x = 5t, y = 4 - t, z = 3 + t$ intersects the line $L_2 : x = 1 + 2s, y = 9 - 3s, z = 2s$ at a point Q. Find Q

$$L_1: \begin{cases} x = 5t \\ y = 4 - t \\ z = 3 + t \end{cases} \qquad L_2: \begin{cases} x = 1 + 2s \\ y = 9 - 3s \\ z = 2s \end{cases}$$

$$\Rightarrow \frac{x-1}{2} = \frac{y-9}{-3} \Rightarrow \frac{(5t)-1}{2} = \frac{(4-t)-9}{-3}$$

$$\rightarrow -3((5t) - 1) = 2((4 - t) - 9) = -15t + 3 = -8 - 2t - 18$$

$$\Rightarrow 13t = 13 \rightarrow t = 1$$

$$\rightarrow \in Q: \begin{cases} x = (5 \times 1) \\ y = 4 - 1 \\ z = 3 + 1 \end{cases} \Rightarrow \begin{matrix} x = 5 \\ y = 3 \\ z = 4 \end{matrix} \longrightarrow Q = (5, 3, 4)$$

Faculty information

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